# Assignment 5

This homework is due *Tuesday* Oct 1.

There are total 41 points in this assignment. 33 points is considered 100%. If you go over 33 points, you will get over 100% for this homework and it will count towards your course grade (up to 115%).

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge.* Your solutions should contain full proofs. Bare answers will not earn you much.

This assignment covers section 3.1 in Bartle–Sherbert.

(1) In this exercise you have to deliver specific inequalities from the definition of the convergent sequence. In each case below, find a number $K \in \mathbb{N}$ such that the corresponding inequality holds for all $n > K$. Give a *specific natural number* as your answer, for example $K = 1000$, or $K = 2 \cdot 10^7$, or $K = 139$, etc. (Not necessarily the smallest possible.)

You can (but you are discouraged to) use a calculator if you want to. However, 1) this problem can be done without using a calculator, 2) even if you do use one, your answers still should easily verifiable without one.

   (a) [2pt] $\left| \frac{890534890.6451}{n} \right| < 0.00019011$

   (b) [2pt] $\left| \frac{100-n}{n} - (-1) \right| < 0.0054352$,

   (c) [3pt] $\left| \frac{200^{10}n + 10^{100}}{n^2 - 10^{200}} \right| < 0.1$,

   (d) [3pt] $\left| \frac{\cos(863n)}{\log n} \right| < 0.032432$,

   (e) [4pt] (See example 3.1.11(d)) $|\sqrt[n]{n} - 1| < 0.01$. *(This item is due Oct 8.)*

(2) REMINDER. Recall that a sequence $X = (x_n)$ in $\mathbb{R}$ **does not** converge to $x \in \mathbb{R}$ if there is an $\varepsilon_0 > 0$ such that for any $K \in \mathbb{N}$ there is $n_0 > K$ such that following inequality holds: $|x - x_n| \geq \varepsilon_0$.

In each case below find a *real number* $\varepsilon_0$ that demonstrates that $(x_n)$ does not converge to $x$.

   (a) [2pt] $x_n = 1 + 0.1 \cdot (-1)^{n+1}$, $x = 1$,

   (b) [2pt] $x_n = 1/n$, $x = 1/2013$,

*— see next page —*

(3) REMINDER. Recall definition of a sequence in $\mathbb{R}$ converging to an $x \in \mathbb{R}$:
Let $X = (x_n)$ be a sequence in $\mathbb{R}$, let $x \in \mathbb{R}$. $X = (x_n)$ converges to $x$ if $\forall \varepsilon > 0 \ \exists K \in \mathbb{N} \ \forall n > K, |x - x_n| < \varepsilon$.

Below you can find (erroneous!) "definitions" of a sequence converging to $x$. In each case describe, exactly which sequences are "converging to $x$" according to that "definition".

(a) [3pt] Let $X = (x_n)$ be a sequence in $\mathbb{R}$, let $x \in \mathbb{R}$. $X = (x_n)$ "converges to $x$" if $\forall \varepsilon > 0 \ \forall K \in \mathbb{N} \ \forall n > K, |x - x_n| < \varepsilon$.
(*If you are confused at this point, think of the problem this way: suppose for some sequence $(x_n)$ and a number $x \in \mathbb{R}$ you know that statement (a) is true. What can you say about about $(x_n)$?*)

(b) [3pt] Let $X = (x_n)$ be a sequence in $\mathbb{R}$, let $x \in \mathbb{R}$. $X = (x_n)$ "converges to $x$" if $\exists K \in \mathbb{N} \ \forall \varepsilon > 0 \ \forall n > K, |x - x_n| < \varepsilon$.

(c) [3pt] Let $X = (x_n)$ be a sequence in $\mathbb{R}$, let $x \in \mathbb{R}$. $X = (x_n)$ converges to $x$ if $\exists \varepsilon > 0 \ \exists K \in \mathbb{N} \ \forall n > K, |x - x_n| < \varepsilon$.

(d) [4pt] Let $X = (x_n)$ be a sequence in $\mathbb{R}$, let $x \in \mathbb{R}$. $X = (x_n)$ "converges to $x$" if $\forall \varepsilon > 0 \ \exists K \in \mathbb{N} \ \exists n > K, |x - x_n| < \varepsilon$.

(4) (Exercise 3.1.8) Let $(x_n)$ be a sequence in $\mathbb{R}$, let $x \in \mathbb{R}$.
(a) [4pt] Prove that $\lim(x_n) = 0$ if and only if $\lim(|x_n|) = 0$.

(b) [3pt] Prove that if $(x_n)$ converges to $x$ then $(|x_n|)$ converges to $|x|$.

(c) [3pt] Give an example to show that the convergence of $(|x_n|)$ does not imply the convergence of $(x_n)$.